Neutrino emission in neutron matter from magnetic moment interactions

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Neutrino emission drives neutron star cooling for the first several hundreds of years after its birth. Given the low energy ($\sim$ keV) nature of this process, one expects very few nonstandard particle physics contributions which could affect this rate. Requiring that any new physics contributions involve light degrees of freedom, one of the likely candidates which can affect the cooling process would be a nonzero magnetic moment for the neutrino. To illustrate, we compute the emission rate for neutrino pair bremsstrahlung in neutron-neutron scattering through photon-neutrino magnetic moment coupling. We also present analogous differential rates for neutrino scattering off nucleons and electrons that determine neutrino opacities in supernovae. Employing current upper bounds from collider experiments on the tau magnetic moment, we find that the neutrino emission rate can exceed the rate through neutral current electroweak interaction by a factor two, signalling the importance of new particle physics input to a standard calculation of relevance to neutron star cooling. However, astrophysical bounds on the neutrino magnetic moment imply smaller effects.

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I. INTRODUCTION

Neutrino physics plays a crucial role in the birth and subsequent evolution of neutron stars, beginning with the hot and dense environment of a supernova [1], where diffusing neutrinos are believed to trigger the explosive event, to the interior of a cold neutron star, whose cooling rate is determined principally by free-streaming neutrinos [2]. While various problems remain with the neutrino-driven supernova explosion mechanism despite recent intensive efforts (see [3] and refs. therein), uncertainties in neutrino emission from neutron stars is limited to a lack of knowledge of the precise underlying equation of state of supra-nuclear matter, where progress is tied to improving many-body calculations [4]. The important point is that the long-term cooling of neutron stars is controlled by neutrino emission, and this stage lasts up to about 10^5 years of age, when cooling by emission of photons becomes more effective.

There is a host of well-known neutrino emission processes that operate in the crust and core of the neutron star. A comprehensive list of neutrino emitting reactions relevant to different regions of the star can be found in [5]. Which one dominates the cooling depends mainly on the temperature and to a lesser extent, on the density. It should be noted that neutron or proton superfluidity can also reduce or enhance neutrino emission dramatically through density-dependent gaps [6]. Since cooling rates are dependent on neutrino emissivities, it has proven useful to turn to neutron star cooling to help identify or constrain new physics contributions to these emissivities [7]. A typical example is the upper bound on the axion mass (or coupling) from axion bremsstrahlung emission in neutron-neutron collisions [8] and stellar cooling [9].

In this work, we are interested in a particular neutrino emission process involving the neutrino magnetic dipole moment. A magnetic moment for the neutrino was postulated almost immediately following Pauli's neutrino hypothesis. The astrophysical consequences of a neutrino magnetic moment, particularly on stellar cooling have been investigated previously (see [10] and refs. 3-9 therein). These works have focused their attention on the plasmon decay process, which is the dominant cooling mechanism in red giants, and also for the crust of a neutron star until thermal conduction lowers temperatures significantly to the point when processes occurring in the core become more efficient. Since the crust is only a small volume fraction of the star, specially for softer equations of state, it is important to address neutrino emission from the core which could also receive important contributions from new physics. As an illustrative example of this, we present a calculation of the neutrino emissivity from neutrino bremsstrahlung in neutron-neutron collisions, where a neutron radiates an off-shell photon (via magnetic moment coupling of the neutron), which subsequently decays to a neutrino-antineutrino pair via magnetic moment coupling. Section IIA outlines the general features and expected significance of particle-physics corrections to known neutrino rates. In section IIB, we explain why neutrino bremsstrahlung from neutron-neutron collisions can be the dominant cooling process in the core, and present the computation of the total emissivity from magnetic moment interactions, which can be appreciable. We briefly discuss corrections to scattering rates of neutrinos by electrons and nucleons which dominantly determine neutrino opacities and mean free paths in supernovae environments. The conditions under

which new particle physics corrections such as presented here can be quantitatively large are explored in section III. We conclude with a discussion of the relevance of our results in section IV.

II. NEUTRINO BREMSSTRAHLUNG THROUGH MAGNETIC MOMENT COUPLING

A. Motivations for neutrino magnetic moment

Neutrino pair production could have new nonstandard channels besides the well known standard model neutral current interactions. We note that for the present analysis, the typical scale of the process involves momentum transfers $\sim$ keV which is also the dominant constraint. Therefore, any new interactions must involve particles with lighter degrees of freedom. In this context, one of the more appealing choices of new physics is to consider a neutrino magnetic moment interaction with the photon field as the mediator.

Within the standard model, neutrinos are massless and chiral and hence have zero magnetic moment. Neutrinos were first introduced by Pauli to explain the energy-momentum conservation in the beta decay process, and can also be assigned a magnetic moment. Carlson and Oppenheimer [11] and later Bethe [12] and Domogatskii et. al [13], considered the neutrino magnetic moment contribution to electron-neutrino scattering. Existence of a non zero magnetic moment can indicate possible masses for the neutrino and hence physics beyond the standard model [14,15]. If the neutrino interaction was purely of the electroweak type, then the neutrino magnetic moment is proportional to its mass and is expected to be very small. In other words, the chiral nature of neutrinos in such a case will impose a small magnetic moment. However, the standard model (which has the electroweak sector as a part of it) is considered to be an effective low energy theory valid at the electroweak breaking scale of 100 GeV. At larger energy scales, new interactions must be introduced and can also lead to values of the magnetic moment larger than the standard model expectations. Results from SuperKamiokande on neutrino anomalies have generated immense interest in the possibility of massive neutrinos, implying physics beyond the standard model [16].

Analogous to a charged fermion, a neutrino can have an anomalous photon-neutrino coupling. This can be defined through an electromagnetic current with coupling strength κ and momentum transfer four vector $q = p' - p$. The current is given by

$$J_\mu = i\kappa(q^2)\bar{u}(p')\sigma_{\mu\nu}q^\nu u(p) \ . \tag{1}$$

Here, $\kappa(q^2)$ is the relevant electromagnetic form factor. The energy dependence can be extracted by taking the coupling to be of dipole form (valid up to a scale Λ) with the form

$$\kappa(q^2) = \frac{\kappa(0)}{(1 + q^2/\Lambda^2)^2} \ . \tag{2}$$

In the present analysis, we shall assume negligible energy dependence; thus, our constraints can be reinterpreted as those obtained for $\kappa(0)$. This approximation is valid so long as the physics that drives J_μ is at a scale below the TeV region. Then the neutrino magnetic moment is the value of the form factor at zero momentum transfer.

Nontrivial electromagnetic effects can be expected in light of massive neutrinos. In this context, a nonzero transition magnetic moment has been used to explain the solar neutrino anomaly observed in terrestrial experiments [17–19]. Furthermore, a nonzero κ has been used to explain the anticorrelation of the solar neutrino flux with the sun spot cycles and the biannual variation of the solar neutrino flux [20]. In the present case, we are interested to know if neutrinos with $\kappa \neq 0$ could as well affect neutron star cooling processes through modified neutrino emissivities. Among the many other conceivable sources for new physics, we find the neutrino magnetic moment to be one of the more attractive examples. The following analysis is to be taken in the spirit of illustrating new particle physics contributions to the existing standard nuclear physics calculations. There could be many other potentially viable options but they are beyond the scope of the present work.

B. The neutrino bremsstrahlung process

For the neutrino emissivity, we will focus on neutrino bremsstrahlung from neutron-neutron collisions alone (pure neutron matter), although similar effects can easily be computed in neutron-proton scattering as well. The reason is that the latter, as well as the standard cooling process, namely the modified Urca ($n+n \to n+p+e^- + \bar{\nu}_e$), is strongly

suppressed due to proton superfluidity, whereas, at the relevant core temperatures, the neutrons are likely to be in the normal phase. This conclusion follows , given that $^1\mathrm{S}_0$ superfluidity of neutrons ceases to exist (due to $s-$wave repulsion above nuclear matter density) and that triplet gaps are much weakened due to strong renormalization of non-central interactions in-medium at or above nuclear matter density [21]. In this case, the dominant neutrino emission process will be neutrino bremsstrahlung in nn collisions. In fact, even at lower densities, where both neutrons and protons pair in the $^1\mathrm{S}_0$ channel, the neutrino pair emissivity can be more effective compared to the modified Urca process [5]. With this motivation, we proceed to estimate the neutrino emissivity from nn bremsstrahlung.

The Feynman diagrams that contribute at tree level to the neutrino pair bremsstrahlung in nn-scattering involve photon emission from each of the four neutron legs, with the photon subsequently coupling to a neutrino pair via the magnetic moment (see eqn.(6) below). For the nuclear matrix element, we use the results of Friman and Maxwell [22] which take into account the long-range one-pion exchange tensor force in the nucleon-nucleon scattering amplitude explicitly and estimate the effects of short-range correlations by cutting off the interaction at short distances and including the short-range rho-exchange tensor force. [1] More recently, revised bremsstrahlung rates in neutron matter that incorporate many-body effects in the medium seem to point to a strong reduction of the emissivity at sub-nuclear and nuclear densities compared to those results [23]. As we are interested only in the relative importance of new particle physics input to the emissivity, we will, for the sake of simplicity, compare our result to the benchmark calculation in [22].

In the notation of [22], the emissivity is given by (for $\hbar = c = 1$)

$$\varepsilon_{\gamma\nu\bar{\nu}} = N_\nu \int \left(\prod_{i=1}^{4} \frac{d^3\boldsymbol{p}_i}{(2\pi)^3} \right) \frac{d^3\boldsymbol{Q}_1}{2\omega_1(2\pi)^3} \frac{d^3\boldsymbol{Q}_2}{2\omega_2(2\pi)^3} (2\pi)^4 \, \delta(E_f - E_i) \, \delta^3(\boldsymbol{P}_f - \boldsymbol{P}_i) \, \frac{1}{s} \left(\sum_{\text{spin}} \left| \mathcal{M}_{nn} \right|^2 \right) \omega_\nu \, \mathcal{F}(E_{\boldsymbol{p}_i}) \, , \tag{3}$$

where $\boldsymbol{p}_i$ denote the momenta of the incoming and outgoing neutrons and $Q_{1,2} = (\omega_{1,2}, \boldsymbol{Q}_{1,2})$ label the neutrino energies and momenta. The delta functions account for energy and momentum conservation, and $\omega_\nu = \omega_1 + \omega_2$ is the total neutrino energy. N_ν denotes the number of neutrino species and $s = 2$ is a symmetry factor for the initial neutrons, when the emission occurs in the final state or vice versa. The function $\mathcal{F}(E_{\boldsymbol{p}_i}) = f(E_{\boldsymbol{p}_1})\, f(E_{\boldsymbol{p}_2})\, (1 - f(E_{\boldsymbol{p}_3}))\, (1 - f(E_{\boldsymbol{p}_4}))$ is the product of Fermi-Dirac distribution functions $f(E) = (\exp(E/T) + 1)^{-1}$, with neutron energies $E_{\boldsymbol{p}_i}$. The matrix element $\mathcal{M}_{nn}$ includes nucleon-nucleon scattering and the coupling to the emitted neutrino pair. The former is described by one-pion exchange,

$$V_{\mathrm{OPE}} = \left(\frac{f}{m_\pi} \right)^2 \boldsymbol{\sigma}_1 \cdot \mathbf{k} \left(\frac{-1}{\mathbf{k}^2 + m_\pi^2} \right) \boldsymbol{\sigma}_2 \cdot \mathbf{k} (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \, , \tag{4}$$

where the pion-nucleon coupling constant $f^2 \approx 4\pi \times 0.08 \approx 1$, and $\boldsymbol{\sigma}$ denotes spin and $\boldsymbol{\tau}$ isospin respectively. $\mathbf{k}$ is the momentum exchanged in the scattering event, equal to $\boldsymbol{p}_1 - \boldsymbol{p}_3$ (or $\boldsymbol{p}_4 - \boldsymbol{p}_2$) . The emitted photon couples to the neutron through the magnetic dipole moment, (denoted by κ_n). Since the neutrons are non-relativistic, and the emitted neutrinos are thermal, to $\mathcal{O}(\boldsymbol{Q}/m_N)$ (where $\boldsymbol{Q}$ is the three-momentum of the neutrino pair), we may write the neutron-photon coupling as

$$i\kappa_n \xi' \frac{\boldsymbol{\sigma} \times \boldsymbol{Q}}{m_N} \xi \, , \tag{5}$$

where ξ, ξ' are incoming and outgoing non-relativistic free nucleon spinors normalized to unity. All other non-relativistic reductions of the fully relativistic neutron-photon vertex couplings are negligible (of higher order in $\boldsymbol{Q}/m_N$ in the limit of small Q^2). In this limit, $\kappa_n = -1.91$ is simply the ratio of the magnetic Sachs factor to the Pauli moment of the neutron. The photon current couples to a neutrino pair also via the neutrino magnetic moment (denoted by κ):

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[1] We have not included the exchange terms and the repulsive short-range rho-exchange, since, as shown in [22], these effects largely offset each other. We have also ignored photon emission from the exhanged pion, because, unlike the intermediate neutron leg, the former does not lead to a small energy denominator. (see eqn.(7))

$$\langle \nu(Q_2)|J^\mu_\gamma|\nu(Q_1)\rangle = i\kappa\bar{u}(Q_2)\sigma^{\mu\nu}(Q_2-Q_1)_\nu u(Q_1)\,, \tag{6}$$

where u's are normalized relativistic spinors for massless neutrinos, and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu\gamma^\nu]$. In computing the matrix element, we have, as in [22], used a non-relativistic approximation for all nucleon propagators, where the lowest term in an expansion in inverse powers of the nucleon mass is retained. In addition, one neglects the neutrino pair energy ω_ν compared to the Fermi energy, since the emitted neutrinos are thermal and have negligible energy compared to the neutrons at typical neutron star densities and temperatures. For the nucleon propagator G, this approximation yields

$$i\,G(\boldsymbol{p}\pm\boldsymbol{Q}, E_{\boldsymbol{p}}\pm\omega_\nu) = \pm i\,\omega_\nu^{-1}, \tag{7}$$

where the positive sign holds if the electromagnetic current is attached to an outgoing nucleon, negative otherwise. The spin trace over the neutrino pieces yields

$$\mathrm{Tr}(l_\mu l_\nu) = -2g_{\mu\nu}s^2 + 8g_{\mu\nu}(Q_1.Q)(Q_2.Q) + s[4Q_{1\mu}Q_{2\nu} + 4Q_{1\nu}Q_{2\mu} + 2Q_\mu Q_\nu] - 4[(Q_1.Q)(Q_{2\mu}Q_\nu + Q_{2\nu}Q_\mu) + Q_1 \leftrightarrow Q_2] \tag{8}$$

where $s = Q^2 = (Q_1+Q_2)^2$ is a kinematic variable (center-of-mass momentum squared). Terms anti-symmetric in μ and ν are dropped since they eventually contract to zero. The non-relativistic reduction of the neutron-photon vertex implies that only the spatial indices (such as $\mu = i, \nu = j$) contribute. Furthermore, by transversality of the photon, the pieces of the neutrino spin summed matrix element that are proportional to Q_i vanish on contraction with the nuclear spin summed matrix element squared, leaving us with

$$\begin{aligned}\langle|\mathcal{M}_{nn}|^2\rangle = 128\frac{\kappa^2\kappa_n^2}{m_N^2\omega^2}\left(\frac{f}{m_\pi}\right)^2\left(\frac{\mathbf{k}^2}{\mathbf{k}^2+m_\pi^2}\right)^2\frac{\boldsymbol{Q}^2}{s}& \\ \times[2(\boldsymbol{Q}_1\cdot\boldsymbol{Q}_2)(\hat{\boldsymbol{Q}}\cdot\hat{\boldsymbol{k}})^2 + 2(\boldsymbol{Q}_1\cdot\hat{\boldsymbol{k}})(\boldsymbol{Q}_2\cdot\hat{\boldsymbol{k}}) - 2(\hat{\boldsymbol{Q}}\cdot\hat{\boldsymbol{k}})\{(\boldsymbol{Q}_1\cdot\hat{\boldsymbol{k}})(\boldsymbol{Q}_2\cdot\hat{\boldsymbol{Q}}) + (\boldsymbol{Q}_1\cdot\hat{\boldsymbol{Q}})(\boldsymbol{Q}_2\cdot\hat{\boldsymbol{k}})\}]\,,&\end{aligned} \tag{9}$$

where an averaging over initial neutron spins has been performed. Note that there is a $1/s$-dependence in the above expression, which indicates an enhancement at small s. There is no divergence, however, since the quantity in the bracket [..] vanishes when the photon is on-shell ($s = 0$); the decay is then kinematically forbidden. Furthermore, when the integrations over the neutrino momenta are performed, an extra factor of s appears in the numerator to cancel the one in the denominator. We may note here that a $1/t$ enhancement does appear in the differential cross-section for the $t-$channel process of electron-neutrino scattering which is related to a forward peak in the differential cross-section [24]. For the differential emissivity, which is a function of the energies of the outgoing neutrinos and the relative angle between them, we have checked that there is a forward peak in the $s-$channel as well, although the $1/s$ enhancement is canceled by the angular phase space measure. Differential neutrino rates are known to be important in the microphysics of neutrino transport in a supernova [25], in which case the forward enhancement in the scattering channel can contribute significantly to thermalization and neutrino opacities. We comment on these toward the end of this section. Proceeding with the calculation for the total emissivity, the neutrino phase space integrals are easily performed by introducing delta functions as

$$1 = \int d\omega_\nu\,\delta(\omega_\nu-\omega_1-\omega_2)\,, 1 = \int d^3\boldsymbol{Q}\delta(\boldsymbol{Q}-\boldsymbol{Q}_1-\boldsymbol{Q}_2)\,, \tag{10}$$

and then using Lenard's identity

$$N_{\alpha\beta} = \int \frac{d^3Q_1}{2\omega_1}\frac{d^3Q_2}{2\omega_2}\,(Q_{1\alpha}Q_{2\beta}+Q_{2\alpha}Q_{1\beta})\;\delta^4(Q_1+Q_2-Q) = \frac{\pi}{12}(Q^2g_{\alpha\beta}+2Q_\alpha Q_\beta)\Theta(Q_0)\Theta(Q_0^2-\boldsymbol{Q}^2)\,, \tag{11}$$

Next, one can decouple the angular parts in the neutron phase space and trade the radial momentum for energy integrals, by restricting the interacting neutrons to the surface of the Fermi sphere (of radius k_{F_n}), since they are strongly degenerate for typical neutron star temperatures. Corrections to this approximation scale as T/E_{F_n} where E_{F_n} is the Fermi temperature (in $k_B = 1$ units). For this purpose, one replaces

$$d^3\boldsymbol{p}_i \rightarrow d^3\boldsymbol{p}_i\,\frac{m_n^*}{k_{F_n}}\,\delta(p_i-k_{F_n})\int dE_{\boldsymbol{p}_i}\,. \tag{12}$$

where m_n^* is the neutron effective mass. After carrying out the angular integrations, we find a compact expression for the emissivity

$$\varepsilon_{\gamma\nu\bar{\nu}} = \frac{A}{(2\pi)^{12}} I_{\nu\bar{\nu}} \left(\frac{m_n^*}{k_{F_n}}\right)^4 \mathcal{S}\,, \tag{13}$$

$$A = \frac{32}{45}\left(\frac{f}{m_\pi}\right)^4 \left(\frac{\kappa\kappa_n}{m_n}\right)^2,\ I_{\nu\bar{\nu}} = \frac{41}{60480}(2\pi)^8 T^8\,, \tag{14}$$

$$S = \int \left[\prod_{i=1}^{4} d^3\boldsymbol{p}_i \delta(p_i - k_{F_n})\right] \left(\frac{\mathbf{k}^2}{\mathbf{k}^2 + m_\pi^2}\right)^2 \delta^3(\boldsymbol{P}_f - \boldsymbol{P}_i)\,. \tag{15}$$

$\mathcal{S}$ is the neutron phase space integral which is facilitated by introducing the integration over momentum transfers through respective delta functions as

$$1 = \int d^3\boldsymbol{k}\,\delta^3(\boldsymbol{k} - \boldsymbol{p}_1 + \boldsymbol{p}_3)\,. \tag{16}$$

The neutron phase space integrals yield [22]

$$\mathcal{S} = 4(2\pi)^3 k_{F_n}^5 \left(\frac{f}{m_\pi}\right)^4 F\left(\frac{m_\pi}{2k_{F_n}}\right), \tag{17}$$

$$F(x) = 1 - \frac{3}{2}\tan^{-1}\left(\frac{1}{x}\right) + \frac{1}{2}\left(\frac{x^2}{1+x^2}\right). \tag{18}$$

It is convenient for purposes of comparison to express the result for the emissivity as a ratio to that obtained from bremsstrahlung via electroweak coupling to the neutrino, whose value $\varepsilon_{\nu\bar{\nu}}$ is calculated in [22]:

$$\frac{\varepsilon_{\gamma\nu\bar{\nu}}}{\varepsilon_{\nu\bar{\nu}}} = \frac{4}{3}\left(\frac{\kappa\kappa_n}{G_F g_A m_n^*}\right)^2. \tag{19}$$

Numerically, we find this ratio to be ($G_F = 1.166 \times 10^{-5}$ GeV^{-2}, $g_A = 1.26, \kappa_n = -1.91$)

$$R^{nn} = \frac{\varepsilon_{\gamma\nu\bar{\nu}}}{\varepsilon_{\nu\bar{\nu}}} = 2.25\left(\frac{\kappa}{10^{-5}\mathrm{GeV}^{-1}}\frac{1}{m_n^*}\right)^2, \tag{20}$$

where m_n^* is taken in units of GeV.

A similar computation for the $np \to np\nu\bar{\nu}$ bremsstrahlung channel which is more important in the stellar crust yields ($\kappa_p = 1.79$)

$$R^{np} = \frac{2}{3}\left(\frac{\kappa}{G_F g_A}\right)^2 \left[\frac{\kappa_n^2}{m_n^{*^2}} + \frac{(1+\kappa_p)^2}{m_p^{*^2}}\right] = 0.31\left(\frac{\kappa}{10^{-5}\mathrm{GeV}^{-1}}\right)^2 \left[\frac{(-1.91)^2}{m_n^{*^2}} + \frac{(2.79)^2}{m_p^{*^2}}\right]. \tag{21}$$

The comparative equations (20) and (21) are also valid for bremsstrahlung from non-degenerate nucleonic matter (the weak interaction rates for which are given in [26]), in the approximation that Pauli blocking of neutrinos can be neglected. Thus, the above estimates hold as one progresses outwards from the neutrinosphere of a young protoneutron star or a supernova as well. In the cores of these objects, neutrinos have short mean free path, and their opacity and thermalization time scale is controlled by (for ν_μ and ν_τ neutrinos) processes such as $\nu-$nucleon and $\nu-$electron scattering. If a neutrino magnetic moment exists, given the $1/t$ enhancement in the scattering channel, one can expect magnetic moment interactions to dominate over weak interactions which do not have this feature in the differential cross-section. This would impact on neutrino transport and equilibration, which is a key component of theoretical models of supernovae explosion. In this work, we have chosen to focus on the emission process (bremsstrahlung) to highlight the possible importance of non-standard contributions to neutrino astrophysics, but we can also outline the effects on scattering processes.

In the cores of protoneutron stars and supernovae, typical ambient temperatures ($T \sim 1 - 50$ MeV) and densities ($\varrho \sim 10^{10} - 10^{14}$ g/cc) imply that neutrinos have short mean free paths due to frequent scattering off electrons and nucleons, which are the dominant sources of neutrino opacity. In both cases, differential rates have been computed,

which serve as the particle-physics input to the Boltzmann equation that in turn evolves the neutrino phase-space distribution ($\mathcal{F}_\nu$). For example, assuming a homogeneous, isotropic thermal bath of scatterers and absorbers, the governing equation for neutrino transport is

$$\frac{\partial \mathcal{F}_\nu}{\partial t} = (1-\mathcal{F}_\nu)J_\nu - \mathcal{F}_\nu A_\nu\,. \tag{22}$$

where J_ν and A_ν are respectively the neutrino emission and absorption currents from one energy bin to another. These quantities are dependent on the differential cross-section for inelastic neutrino scattering. However, since we are only interested in the relative importance of the magnetic moment effects, we may compare elastic differential cross-sections for simplicity. This is equivalent to assuming a redistribution of neutrinos only in space and not in energy. For a rigorous treatment of inelastic rates and their incorporation into the Boltzmann equation, one can follow the structure function formalism developed in [29]. Below, we will only compare elastic rates as they will suffice to estimate the magnitude of the corrections through magnetic moment interactions.

For neutrino-nucleon ($N = n$ or p) scattering, the differential rates for elastic scattering as a function of the angle between the incident and scattered neutrino θ and the neutrino energy E_ν are given by

$$\begin{aligned}\frac{d\sigma^{\nu n\to\nu n}}{d\cos\theta} &= \frac{G_F^2 E_\nu^2}{2\pi}\bigg((1+\cos\theta) + g_A^2(3-\cos\theta)\bigg)\,, \qquad (23)\\ \frac{d\sigma^{\nu p\to\nu p}}{d\cos\theta} &= \frac{G_F^2 E_\nu^2}{8\pi}\bigg((16\alpha^2-8\alpha+1)(1+\cos\theta) + g_A^2(3-\cos\theta)\bigg); \quad \alpha = \sin^2\theta_W = 0.23\,.\end{aligned}$$

which can be compared against the result from the magnetic moment interaction

$$\begin{aligned}\frac{d\sigma^{\nu N\to\nu N}}{d\cos\theta} &= \frac{\beta_N^2\kappa^2 E_\nu^4}{2\pi\ E_N^2}\sin^2\left(\frac{\theta}{2}\right)\,, \qquad (24)\\ \beta_N &= \frac{\kappa_n}{m_n^*}\ \text{for neutrons,} \quad \frac{1+\kappa_p}{m_p^*}\ \text{for protons}\,.\end{aligned}$$

Assuming thermal neutrinos with an energy $E_\nu = T$ (in $k_B = 1$ units), a nucleon effective mass of 900 MeV and $\kappa = 10^{-7}\,\mu_B$, the ratio (denoted $R_{n/p}$ respectively) of the magnetic moment induced scattering rate to the electroweak rates is displayed as a function of the scattering angle in the figure below. We employ two sets of typical temperatures, densities, nucleon and electron fractions obtained in a one-dimensional core-collapse model, Star [26]. We have scaled the proton ratio R_p at the higher temperature of $T = 10.56$ MeV by a factor 1/10 to fit within the graph. It is evident that the magnetic moment corrections are more important in the case of neutrino scattering off protons, particularly at temperatures of 5 MeV or more.

For neutrino-electron scattering, the elastic differential cross-sections ignoring polarization effects of the medium are given by

$$\frac{d\sigma_{EW}^{\nu e^-\to\nu e^-}}{d\cos\theta} = \frac{G_F^2}{2\pi\ s}\frac{(s-m_e^2)^2}{(1-t/M_Z^2)^2}\,, \quad \frac{d\sigma_{MAG}^{\nu e^-\to\nu e^-}}{d\cos\theta} = \frac{\alpha_e\kappa^2}{2}\frac{(s-m_e^2)(m_e^2-s-t)}{st}\,. \tag{25}$$

where $s = (E_\nu + E_e)^2, t = -2E_\nu^2(1-\cos\theta)$. A comparison of these rates, denoted by the curve R_e, is also presented in the same figure, for $T = 4.52$ MeV. There is a sharp forward peak which comes from the enhancement in magnetic moment scattering at small momentum transfer (or small angles). This leads to a logarithmic divergence in the total cross-section $\sigma_{\rm tot}^{MAG} \sim \alpha_{\rm em}\kappa^2\log(t_{\rm max}/t_{\rm min})$, which can be tamed by choosing typical finite cutoffs, for eg. $t_{\rm max} \sim$ MeV, $t_{\rm min} \sim$ keV. Although these total cross-sections are small compared to the electroweak cross-section $\sigma_{\rm tot}^{EW} \sim G_F^2 s/\pi$, and we have largely ignored the fact that the nucleons/electrons experience strong medium effects, these estimates point to significantly increased differential rates for forward scattering (and possible energy redistribution from inelastic rates) upon inclusion of magnetic moment effects for neutrino diffusion in supernovae.

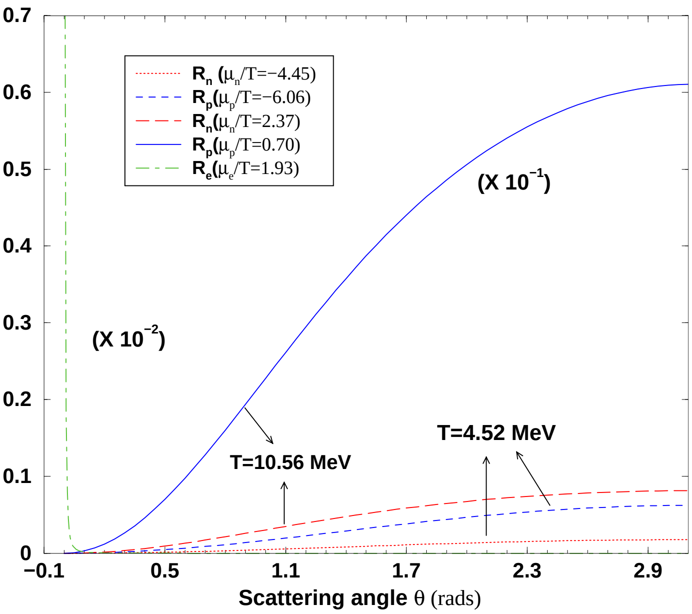

FIG. 1. Ratio of differential rates from magnetic moment interactions ($\kappa = 10^{-7}\ \mu_B$) to electroweak, for $\nu - n$ and $\nu - p$ scattering. The $\nu - p$ ratio at $T = 10.56$ MeV is scaled by 0.1 to fit within the plotted range. The dot-dashed line is the ratio, scaled by a factor .001, in case of $\nu - e^-$ scattering (peaked at $\theta = 0$) for $T = 4.52$ MeV. The zero on the x-axis is slightly offset to show the peak.

III. CONTRIBUTION TO NEUTRINO BREMSSTRAHLUNG EMISSIVITY DUE TO κ

We can assess the importance of these effects by utilizing the upper bounds on the e, μ, and τ neutrino magnetic moments coming from collider experiments and astrophysical data on stellar cooling. The process $e^+e^- \rightarrow \nu\overline{\nu}\gamma$ has been used to examine the constraints on the photon induced electromagnetic interaction for the neutrino (κ). Earlier, bounds for κ were obtained at the Z pole [30] and also below the Z pole [31]. Currently, both the ν_e (from SN1987A data) and ν_μ (from $\nu_\mu e^-$ scattering experiments) set strong bounds on κ, limiting it to less than $10^{-10}\mu_B$. Using the solar neutrino data from Kamiokande, a bound on κ ($\sim 10^{-10}\mu_B$) was obtained where an energy independent suppression of the ν_e flux was assumed [32]. Such an approach is now ruled out considering the fact that the ν_e flux depletion is indeed energy dependent. Assuming three active neutrino flavors, as suggested by the Z width measurements at LEP, only the ν_τ is yet not so strongly constrained with respect to its anomalous electromagnetic couplings. Recently, the atmospheric neutrino data from Super-Kamiokande has been used to achieve bounds on ν_τ magnetic moment ($\kappa \sim 10^{-7}\mu_B$) assuming maximal $\nu_\mu \leftrightarrow \nu_\tau$ mixing [33] and is three orders of magnitude weaker than the collider bounds. However, given that the mass of the ν_τ is not expected to be very different from other neutrino flavors, one could extrapolate the astrophysical limits (which are flavor blind) to the case of ν_τ as well. In this case, we obtain values of $\kappa \leq 10^{-10}\mu_B$ [34]. For the present purposes, we therefore consider the range $10^{-10}\mu_B \leq \kappa \leq 10^{-7}\mu_B$ and we shall show how this variation can affect the relative contribution to the neutron star cooling from standard expectations. To illustrate, we use the following conversions: $1\mu_B = 5.8\cdot 10^{-9}$ eV/Gauss and (ii) 1Gauss $= 7\cdot 10^{-2}\text{eV}^2$ and therefore, $1\mu_B = 8.2\cdot 10^{-8}\ \text{eV}^{-1}$. Using (21) and setting the nucleon masses to be of 0.9 GeV, we obtain the following limits:

$$1.86\cdot 10^{-6} \leq R^{nn} \leq 1.86, \quad 3.61\cdot 10^{-6} \leq R^{np} \leq 3.61 \text{ for the range } 10^{-10}\mu_B \leq \kappa \leq 10^{-7}\mu_B\ . \tag{26}$$

Clearly, the upper limit indicates that the new physics can alter the standard model-based result for the bremsstrahlung emissivity by a factor of 2-3 while the more conservative lower limit will have negligible effects and we may ignore any particle physics corrections to standard results. The important point is that we cannot rule out sizeable corrections to neutrino bremsstrahlung rates from physics beyond the standard model.

IV. CONCLUSIONS

We have explored the relevance of particle physics beyond the standard model to standard neutrino pair emission rates from neutron star interiors, focusing especially on the role of the neutrino magnetic moment. General arguments based on relevant energy scales imply that a neutrino magnetic moment provides the most interesting and feasible correction to known physics. In the low-energy approximate treatment of neutrino pair bremsstrahlung from neutron matter, we have estimated the correction to a benchmark calculation of the neutrino pair emissivity from electroweak interactions in neutron matter. The magnitude of the correction is estimated from limits on the neutrino magnetic moment κ.

While stringent collider bounds on the ν_e and ν_μ electromagnetic moments exist, the ν_τ is not as nearly strongly constrained, and employing a range set by collider and astrophysical inputs, we find that the standard emission rates may be revised by upto a factor of two or more. We expect similar results for neutrino scattering via magnetic moment interactions, leading to modified neutrino opacities. We have estimated the magnitude of such corrections by using elastic rates and free scatterers. For practical calculations in supernovae physics, we need to explore similar corrections to inelastic rates, which requires including medium effects and energy transfer. These studies are currently in progress. The magnetic moment interaction vertex remaining unaffected in medium, we would expect comparable enhancements upon inclusion of medium effects, therefore we have, in this work, presented ratios of rates rather than their absolute values. We observe that large corrections to the differential cross-section are obtained for neutrino-proton scattering over a wide range of angles and neutrino-electron scattering in the forward direction. The correction to neutrino-neutron scattering is expected to be at the few percent level.

It is also pertinent to mention here the consequences of these revised emissivities for the temperature versus time profile (cooling curve) of a typical neutron star. While stellar cooling is determined (for late times) principally by neutrino emission from the core, a temperature profile requires mapping the surface temperature to the core temperature, a procedure that is strongly dependent on important factors such as local surface temperature variations, magnetospheric emission as well as the composition of the stellar envelope and atmosphere. These details form an integral part of the surface temperature determination. Furthermore, knowledge of the temperature gradient within the core and upto the star's surface requires as input, the equation of state of nuclear matter at supra-nuclear density, and knowledge of related variables such as the incompressibility and the symmetry energy, which are poorly constrained in neutron matter. Model-dependent theoretical uncertainties tend to mask the effects of improved or revised estimates of standard emissivities when incorporated into a numerical simulation of neutron star cooling, and observational difficulties complicate the direct comparison of such numerical results to data. This uncertainty may be removed if a novel temperature or density dependence is discovered as in the case of the direct urca process or if exotic phases such as pion or kaon condensates exist from which cooling can be more efficient. It is unlikely that particle physics corrections to emissivities that are of $\mathcal{O}(1)$ would considerably alter the overall cooling scenario. Nevertheless, the interesting possibility remains that large contributions from non-standard model physics may exist and yet remain hidden in the microphysics of neutron star cooling.

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